now considering. The only sure method of procedure in such a case seems to be a discussion for magnitude equation plate by plate, which is impossible unless the published material is amplified.

16. *Conclusion.*—The results of this discussion make it clear that the star places published in the Paris Circulars are by no means homogeneous, and that errors exist in some of the series which, uneliminated, would ruin the determination of the solar parallax. I have, however, full confidence that these difficulties can be overcome. By the great kindness of the directors of several observatories, the separate results from each of their plates have recently been placed at my disposal, and the discussion of this material is now proceeding at Cambridge, with results which I hope to communicate to the Society.

17. But our conclusions have, it seems to me, an interest wider than that of the particular problem in hand. Most of the photographic telescopes concerned have been engaged for years upon the Astrographic Chart and Catalogue. With that experience behind them, they undertook shares in the Eros co-operation; and several of the resulting series of star places are affected by errors much larger than are accounted tolerable in the Astrographic Catalogue—a disquieting result.

18. The work summarised in this paper was done concurrently with that described in paper No. 4, and the same acknowledgments are due to the Government Grant Fund of the Royal Society; and to Miss Julia Bell, who has carried out the greater part of the computation.

Cambridge Observatory:
1906 *November* 7.

On the Distribution of Energy in the Continuous Spectrum. By E. T. Whittaker, Sc.D., F.R.S., Royal Astronomer of Ireland.

The distribution of energy in the continuous spectrum of the "black body" at various temperatures, which is of interest astronomically from its application to the problem of solar and stellar temperatures, has been studied from the experimental side by many physicists, and the results obtained have been co-ordinated into the form of empirical laws of radiation. One such empirical law, which is due to Lummer and Pringsheim,* and closely represents the observations, is to the effect that the intensity in the part of the spectrum at wave-length λ, radiated by a black body at the absolute temperature T, is proportional to

$$T\lambda^{-4}e^{-\frac{c}{(\lambda T)^{1\cdot 3}}} \quad . \quad . \quad . \quad . \quad \text{(I.)}$$

where c is a constant.

The mode of occurrence of T in this formula is not really empirical, as it can be proved by thermodynamical considerations

* *Deutsch. Phys. Gesells. Verh.*, 2, 12*a*, 103–180 (1900).

that if the energy between wave-lengths λ and $\lambda + d\lambda$, when the body is at temperature T_0, is $\psi(\lambda)d\lambda$, then the corresponding quantity when the body is at temperature T is

$$\left(\frac{T}{T_0}\right)^5 \psi\left(\frac{T\lambda}{T_0}\right) d\lambda \quad . \quad . \quad . \quad \text{(II.)}$$

Lummer and Pringsheim's formula is evidently constructed to satisfy this condition, and what is really empirical in the formula is therefore the mode of occurrence of λ when T is constant; in other words, the distribution of energy in the spectrum at some one definite temperature is taken from observation, and the distribution of energy in spectra at all other temperatures can then be deduced from thermodynamical principles.

For the above formula (I.) no theoretical justification has yet been found. But one feature, common to it and to all the rival formulæ which have been suggested since the results of observations in the extreme infra-red have been available, is that the distribution of energy in the region of long wave-lengths is proportional to

$$T\lambda^{-4} d\lambda \quad . \quad . \quad . \quad . \quad \text{(III.)}$$

In other words, the curve of intensity I in the spectrum must approximate to the curve $I = CT\lambda^{-4}$ in the ultra-red where C is a constant.

This result (III.) may now be regarded as a well-established result of observation;* and it becomes important to inquire whether it can be explained on theoretical grounds.

Several writers have discussed this question by the aid of assumptions regarding the nature of the radiating mechanism in the black body. Lord Rayleigh † suggested the application of the Boltzmann-Maxwell doctrine of the partition of energy among the different modes of vibration. The difficulty here lies in the doctrine itself, which is not free from uncertainties, and would give results inconsistent with observation if applied to the shorter wave-lengths.

Planck ‡ has attacked the matter from the point of view of a distinctive theory of the mechanism of radiation. The radiating body is supposed to contain a great many electrical vibrators, each having its own period of free vibration, and exchanging energy with the æther and the material molecules. By discussion of such a system, Planck derives the law of radiation

$$\frac{c_1 \lambda^{-5}}{e^{\frac{c_2}{\lambda T}} - 1} d\lambda,$$

* It is further confirmed by the observations of Rubens and Kurlbaum, *Ann. d. Phys.*, iv. p. 649 (1901).

† "Remarks upon the Law of Complete Radiation," *Phil. Mag.*, xlix. 539 (1900). Lord Rayleigh's paper was written before the accuracy of the law had been experimentally verified, and therefore constitutes a successful prediction from theory. See also Jeans, *Phil. Mag.* (6) x. 97 (1905).

‡ Ueber das Gesetz der Energieverteilung im Normalspectrum, *Ann. d. Phys.*, iv. 553 (1901).

where c_1 and c_2 are constants. This formula evidently satisfies law (III.), and is indeed in good accord with observation for the shorter wave-lengths also.

Later, Lorentz* gave a different view of the mechanism of radiation: he explains the emission of a metal by means of the heat-motion of its free electrons, which are regarded as moving to and fro with a velocity of agitation increasing with the temperature, and frequently striking against the molecules. He shows that such a system would emit the longer radiations in accordance with formula (III.) above.

The object of the present paper is to show that formula (III.) can be established on theoretical grounds quite apart from any assumptions as to the mechanism of radiation; that in fact it is a necessary consequence of the laws of thermodynamics, together with the usual assumptions regarding the nature of the white light.

Natural radiation is now generally understood to consist of a succession of discrete disturbances or "pulses" in the æther, which are not co-ordinated as regards phase, and each of which consists of compensating positive and negative parts, so that the curve representing a pulse has the total area of those portions of it which are below the axis equal to the total area of those portions which are above the axis. By the agency of a prism or grating, a single pulse of this kind is drawn out into trains of periodic disturbances, the dispersive apparatus in fact performing a resolution of the pulse which corresponds to that furnished analytically by Fourier's integral; and it is this resolution which constitutes spectroscopy.

Suppose, then, that a pulse in æther is represented by $f(x-ct)$, where x denotes distance measured in the direction of propagation of the pulse, t denotes time, and c is the velocity of light in æther. We shall express the discrete character of the pulse by supposing that $f(x-ct)$ is zero except when $x-ct$ lies between the limits a and b.

Then Fourier's resolution can be written

$$f(x-ct)=\frac{1}{\pi}\int_0^\infty dn\int_a^b \cos\{n(x-ct-\mu)\}f(\mu)d\mu$$

So when the pulse is spectroscopically resolved, the elements with wave-lengths between λ and $\lambda+d\lambda$ in æther will be (writing $2\pi/\lambda$ for n, and also for convenience writing $a+y$ for μ)

$$\frac{2d\lambda}{\lambda^2}\int_0^{b-a}\cos\left\{\frac{2\pi}{\lambda}(x-ct-a-y)\right\}f(a+y)dy$$

Now throughout the range over which the integration is taken, y is small compared with λ if the wave-length is taken so far in the infra-red that λ is large compared with the extent of the pulse. We can therefore expand the cosine in ascending powers of $\frac{y}{\lambda}$ and

* "On the Emission and Absorption by Metals of Rays of Heat of great Wave-lengths," *Proceedings of the Amsterdam Academy of Sciences* (English edition), v. 666 (1902).

retain only the leading terms of the expansion. Retaining for the present the first two terms, the preceding expression becomes

$$\frac{2d\lambda}{\lambda^2}\cos\left\{\frac{2\pi}{\lambda}(x-ct-a)\right\}\int_0^{b-a}f(a+y)dy$$

$$+\frac{2d\lambda}{\lambda^2}\frac{2\pi}{\lambda}\sin\left\{\frac{2\pi}{\lambda}(x-ct-a)\right\}\int_0^{b-a}yf(a+y)dy$$

The first of these integrals vanishes in consequence of the condition that the pulse consists of compensating positive and negative parts; the second integral depends on the particular form of the pulse, but is independent of λ, and will in general have a finite value different from zero, which we shall denote by C. The spectroscopic element of the pulse with wave-lengths between λ and $\lambda+d\lambda$ is therefore

$$\frac{4\pi C d\lambda}{\lambda^3}\sin\left\{\frac{2\pi}{\lambda}(x-ct-a)\right\}$$

Now if in any disturbance the spectroscopic element with wave-lengths between λ and $\lambda+d\lambda$ is

$$f(\lambda)\,.\,d\lambda\,.\,\sin\left\{\frac{2\pi}{\lambda}(x-ct-\epsilon)\right\},$$

it is known * that the energy radiated in this interval is proportional to $\lambda^2\{f(\lambda)\}^2 d\lambda$. So in the present case the energy radiated with wave-lengths between λ and $\lambda+d\lambda$ is proportional to $\lambda^{-4}d\lambda$. As the various pulses are supposed to be entirely unco-ordinated as regards phase, this law which holds for each of them individually will hold also for their aggregate; and therefore, in the radiation emitted by any body, the energy in the part of the spectrum between λ and $\lambda+d\lambda$ is proportional to $\lambda^{-4}d\lambda$ in the region of longer wave-lengths.

From this result, by an application of the thermodynamical theorem (II.) above, we immediately deduce the consequence that the radiation of a body at temperature T is, in the ultra-red, proportional to $T\lambda^{-4}d\lambda$.

Law (III.) is thus established as a direct consequence of the nature of white light, without reference to the mechanism of radiation in the radiating body.

On the Resolving Power of Spectroscopes. By E. T. Whittaker, Sc.D., F.R.S., Royal Astronomer of Ireland.

The resolving power of a spectroscope is, according to the usual definition, the value of $\frac{\lambda}{\delta\lambda}$, where $\delta\lambda$ is such that the luminous centre of the spectral line of wave-length $\lambda+\delta\lambda$ falls on the first minimum of intensity of the line of wave-length λ. It is a well-

* *Cf.* Lord Rayleigh, *Phil. Mag.*, xxvii. 460 (1889).